

SENIOR MATHEMATICS COMPETITION 2012

Preliminary round Thursday 17th May 2012

*Solutions*

*Total: 52 marks*

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**SENIOR MATHEMATICS COMPETITION 2012**

[3

Marking schedule.

2. A three digit number is equal to the sum of the cubes of its digits. Find all such three digit numbers. 153, √ (one number) 370, 371, 407 √√ (the other three) [3]

1. Find the number between 1 and 108 which is first alphabetically (i.e. would be first in the dictionary). Eight√ √ [2] [2]

x

x

1

1

x

4. Alice, Bob and Carol duel with pistols. Alice is a poor shot and hits 1/3 of the time. Bob is better and hits 2/3 of the time. Carol is a sure shot. The shooters take turns to fire one shot at either of the other two: Alice then Bob then Carol then Alice then Bob … until one person remains, uninjured. What is Alice’s best strategy, and why? Shoot in the air. √ P(A remains) = 1/3; if A shoots at B or C P(A remains) is less than 1/3. (working)√√√ Show working to get P = 1/7 if A shoots at B or C.√[5]

1. An equilateral triangle is inscribed in a circle. A chord cuts the triangle so that the length of the chord is two cm more than half the side of the triangle and the small triangle formed is also equilateral with each side half that of the original sides. (See diagram above) Find the lengths of the original sides. (i.e. 2x) x2=(x+1).1 √√ so 2x = 1 + √5 √ [3]

b

A

D

B

C

R

b

1. For kite ABCD with angles a,b,c and b respectively, (see diagram above) if length AC is R, then show that the area of the kite is equal to [ R2sin(a/2)sin(c/2)]/sin((a+c)/2). Sine rule, symmetry √√√[3]

x

x

2x

7. Draw a right angled triangle with hypotenuse labelled h and one other side of length 2. From the midpoint of the side of length 2, draw a segment to meet the opposite vertex. Label the angle between this median and the original hypotenuse “ x”. Given that x is a maximum when h(h2 – 2) = 2h(h2 – 4), find the values of h and x which give a maximum for x, and show clearly **without using calculus** that it is a maximum value. h = 0 giving a contradiction or h2 = 6, h =√6 √, ignore negative √ so tan x = √2/4 and x = 19.4710 √ if h = √5 tan x = 1/3 and x = 18.4350, if h = √7 tan x = √3/5 and x = 19.1070 so max. √√ [5]

1. Find values for x and y so that log572 = xlog548 + ylog554. 72 =( 24x3)xx(22x33)y √ = 24x+2y x 3x+3y√ so 4x+2y = 3 and x+3y = 2 √ so x= ½ , y = ½ √ [4]

12.A circle of radius 1cm and a circle of radius 3cm have centres 10cm apart. If X is a point on one circle and Y is a point on the other, describe thoroughly the set of points M where M is the midpoint of XY.

Diagram would help. Locus is the area between two circles centred on the midpoint of the segment joining the centres of the original circles, with the outer circle: radius 2cm, the inner: radius 1cm. √√√√ [4]

Total = 52 marks

11.(a) Eleven “mixed number” combinations of all nine non-zero digits give 100. Eg.81+5643/297 = 100. Find five of the other ten. 3+69258/714, 81+7524/396, 82+3546/197, 91+5742/638, 91+5823/647, 91+7524/836, 94+1578/263, 96+1428/357, 96+1752/438, 96+2148/537 One mark each. [5]

(b)Nine multiplications each use all the non-zero digits once only. Eg. 42x138 = 5796. Find four of the other eight.4x1738=6952, 4x1963=7852, 12x483=5796, 18x297=5346, 27x198=5346, 28x157=4396, 39x186=7254, 48x159=7632.One mark each. [4]

10. We walked at 4kph on the flat, 3kph uphill and then 6kph back downhill and 4kph on the flat again.If we started at 3pm and returned at 9pm.(a) How far did we travel? (b) Within half an hour when were we at the top of the hill?(a) 24km √ to go and return 1km takes 30 min √ since on flat it is 2x15min and on hill it is 10min +20 min √ (b) 6:30pm √ working to show times if all flat or all hill√ [5]

9. A function f(a,b) is defined by f(a,b) = a if a=b, f(a,b) = f(a-b,b) if a>b, f(a,b) = f(b-a,a) if b>a.

Find (a) f(17,9), (b) f(18,12), (c) f(8,12), (d) what does this function do?

1. 1 √ (b) 6 √ (c) 4 √ (d) finds HCF√ [4]

8. A square courtyard has 20 doors on each side, which contains 21 equal parts. They are all numbered round, beginning at one corner. From which one of door number 9, 25, 52, 73 is the sum of the distances to the other three doors the least? Show working **without** calculating any decimal places.

Label doors A,B,C,D. Then AB =√(122+52) = 13, AC = 21, AD =√(92+82) =√145 >12, BC = √(162+122) = 25, BD = √(32+212) = √450 > 21, CD = √(92+132) = √250 > 15 √√√. So distances from A: 46 to 47, B: 54 to 55, C: 56 to 57, √ D: 48 to 51 (not 48 to49)√ Hence shortest from A = 9.[5]